Technical Publications

Ampère's Law Unified

Theory

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	Clarification: equation 13a

THE PROBLEM

Ampère’s law is very well known, and is a critical part of the foundation of electromagnetic theory. It describes and quantifies the relationship between an electric current and the magnetic field it generates.

In it’s original form it only recognised flowing current passing through, and generating a magnetic field around, an arbitrary 2-dimensional loop. But this theory was found to be incomplete and was later modified by James Clark Maxwell to recognise the effect of so-called displacement current. This amended version is properly known as the Ampère-Maxwell law.

The amended law is usually shown as a formula:

1. $\oint B.\delta l=\mu(I+i_d)$

Where

B is the magnetic field
$\oint$ represents the sum around a 2-dimensional closed loop of arbitrary shape
δl represents elements of the route around the loop
μ is the magnetic permeability, which varies according to the material of the environment
I is the total current flowing perpendicularly through the loop
i_d is the so-called displacement current

It is important to note that any current flowing anywhere except through the loop has no effect. Also, any current that passes through the loop at an angle other than the perpendicular has diminished effect according to the cosine of the variation.

The problem is that this displacement current does not actually exist as such. It is a fudge to account for the magnetic field that is generated around an insulator when electric charges are moved around it. In other words it is a phantom current that appears to flow through an insulator.

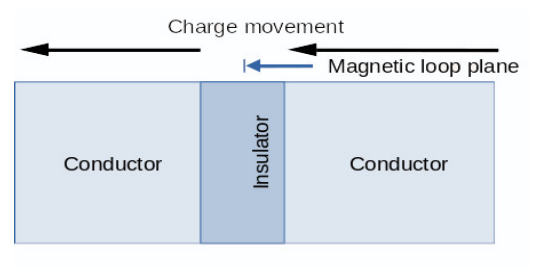
For many practical applications the effect is so small as to be entirely insignificant and is generally disregarded. However when studying electromagnetic waves and their propagation this displacement current is fundamentally important: electric current cannot flow where there are no mobile electrons, and without this displacement current a critical link connecting the electric and magnetic oscillations would be absent.

It would therefore appear that there are two distinct phenomena at work here. One is the effect of a steady electric current, and the other concerns the motion of electric charges in the vicinity.

This doesn’t feel at all right. Surely there has to be a unifying phenomenon that underpins both these effects.

DISPLACEMENT CURRENT

In this diagram an insulator is placed between two conducting plates, and a current is used to charge it as if it were a capacitor.



Current in an insulator

According to the amended Ampère’s law, there is a displacement current seemingly passing through the insulator, but only when the capacitor is being charged or discharged. This results in a magnetic field, which would be edgewise-on in the diagram and not shown, and has been verified by experiment. If the current is stopped, leaving an electric charge in place, it is found that the magnetic field collapses.

While an insulator has no mobile electrons it can still propagate an electric flux and therefore contain an electric field. So it would appear that the electric field or, more likely the rate of change of the electric field, created by the surface charges is generating the magnetic field.

In the simple case where the insulator has parallel plates that are closely spaced, the electric field is directly related to the amount of charge.

2 $E=Q/\varepsilon$

Where

E is the total electric field, spread over the entire area
Q is the accumulated charge
ε is the permittivity of the insulator material

But, since Q is the sum of electron movements, the rate of change is the current at any moment, so

3 $dQ/\delta t=i_d$

Where

dQ is the incremental change in charge
δt is the corresponding incremental change in time
i_d is the instantaneous charging/discharging current

So from 2 and 3

4 $\delta E/\delta t=i_d/\varepsilon$
4a $i_d=\varepsilon.\delta E/\delta t$

When applied to the revised Ampère’s law this current is the displacement current, so the law can be rephrased using a real electric field instead of a non-existent phantom current:

5 $\oint B.\delta l=\mu(I+\varepsilon.\delta E/\delta t)$

This is not unlike published theory that uses a so-called displacement field to account for the electric field, but the above is perhaps a clearer derivation that doesn’t require unfamiliar phenomena to be invented. However it takes no account of the behaviour of the insulator. That may introduce second-order or greater effects, and perhaps even a time delay due to the absorption of energy, which may affect high frequency or high power characteristics.

While the above rephrases Ampère’s law it doesn’t do much to unify the required components of current. So it is necessary to examine the current along a conductor.

CURRENT

We think of electric current as a bulk phenomenon, where charge flows continuously under the influence of a potential difference. So the concept of electric field, while fundamental to the motivation of electrons to move about, is rarely considered.

When a current passes along a wire, there is no apparent electric field. Indeed, if a higher voltage were used to pass current through a bigger resistance, any electric field would necessarily also be bigger, but the current could be the same and the magnetic effect could also be the same. So there is no obvious connection between an electric current and an associated electric field that may be responsible for the magnetic effect.

To understand how this might work we need to step away from the bulk viewpoint and see the individual moving charges.

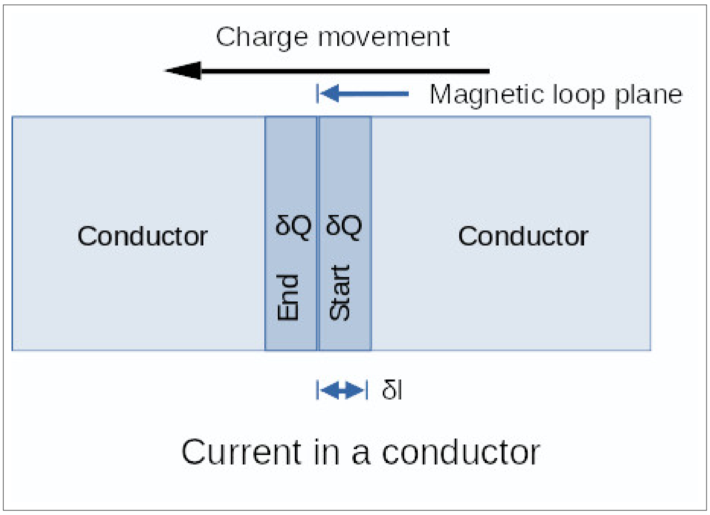
Electrons are normally trapped in atoms and their electric fields balanced by the opposing charges of an equal number of protons. But when there is an electric current something causes the electrons to break free of their energy traps and migrate. This can only be an electric field. The strength of the fields within a conductor therefore has to exceed the energy traps, which vary from one element to another. So different elements seem to require different internal electric fields to generate the same current. But because the subatomic particles are so closely bound within the atoms the electric fields are not detectable outside the conductors.

When a current flows the electrons, together with their individual electric fields, are moving. But the nuclei, with their opposing electric fields, are not. So if we are looking for a changing electric field it would seem that it does exist even though there is no detectable electric field causing it. This is different from the displacement current situation, where there is a real and measurable electric field that is changing.

Another way of viewing this is to imagine a very thin slice through the conductor in the plane of the notional magnetic loop.

When an electron passes across this slice the effect is to reverse the electric flux passing across the slice and therefore the field acting on the slice. It is the same as if the electron were extracted sideways completely out of the loop and replaced sideways slightly further along the conductor. It appears that this would have the same effect as the displacement current (in which the electric charges are moved through an external circuit that is outside the loop) and result in the same magnetic effect. This only occurs when there is a flow of current, and is proportional to that current.

Mathematically this can be calculated from the effect of migrating a small slug of electrons across a boundary within a conductor.



In the diagram a quantity of electrons in the slug is moved in it's entirety across the boundary representing a magnetic loop seen edgewise-on.

The total charge in the slug is found from the volume, mobile electron density and electronic charge.

6 $dQ=a\rho q\delta l$

Where

- dQ is the total charge in the slug
- a is the cross-section area of the conductor
- ρ is the charge carrier (mobile electron) density
- q is the charge of each electron
- δl is the slug length

If the slug is very thin the electric flux from the notional end surface of the slug is half the total charge, so from standard electric field theory

7 $F=dQ/2\varepsilon$

Where

F is a vector electric flux.
ε is the electric constant

If the area of the conductor is much greater than the atomic size the flux is effectively orthogonal to the notional end surface of the slug. So the total electric field E at the surface of the slug is the same as the flux and can be found from 6 and 7.

8 $E=a\rho q\delta l/2\varepsilon$

If the entire slug moves by it's own length the electric field at the original surface is reversed, so the change to the field at that surface is twice the field itself. So if this takes place in time δt the effective rate of change of the electric field can be found.

9 $\delta E/\delta t=a\rho q\delta l/\varepsilon\delta t$

The electric current I is the bulk rate of flow along the conductor. So from 6

10 $I=dQ/\delta t=a\rho q\delta l/\delta t$

And from 9 and 10

11 $I=\varepsilon\delta E/\delta t$

Which is the same as 4a describing the displacement current.

UNIFICATION

From the foregoing we can now represent the two components of the Ampère-Maxwell law entirely in terms of a single natural phenomenon, which is the rate of change of a single, combined electric field.

From 4a and 11

12. $\oint B.\delta l=\mu\varepsilon(\delta Ec/\delta t+\delta Es/\delta t)$

Where Ec is is a function of the internal electric fields in a conductor and Es is the electric field in a non-conductor. Both are vectors.

Both vector fields are only significant for their component orthogonal to the plane of the notional magnetic loop. So they can be added and combined with the z axis unit vector (ż) of the magnetic loop.

13 $\oint B.\delta l=\dot{z}.\mu\varepsilon\delta(Ec+Es)/\delta t$
or
13a $\oint B.\delta l=\dot{z}.\mu\varepsilon\delta\Sigma E/\delta t$

Where ΣE is the total electric field from all sources

Unless it is found that this theory has been published elsewhere it could perhaps be known as the Ampère-Maxwell-Grossi law

This equation can be directly correlated with the Ampère-Maxwell equation with the following relationships.

14 $\varepsilon\delta Ec/\delta t=I$
15 $\varepsilon\delta Es/\delta t=\delta Q/\delta t=i_d$

SO WHAT?

By showing that a magnetic field is generated simply by the rate of change of an electric field it perfectly mirrors the inverse relationship where an electric field is generated by the rate of change of a magnetic flux (Faraday’s law). No fudge factors or phantom phenomena need to be invented.

In it’s published form the Ampère-Maxwell law fails to clearly reveal the symmetrical relationship between electric and magnetic fields.

